Randomized Numerical Linear Algebra
MATH-GA 2012
Spring 2022

Course Syllabus

Cross listing	CSCI-GA 2945
Lecture	Monday 11:00 AM -12:50 PM CIWW 1302
Instructor	Mike O'Neil CIWW 1119 and 2 MTC 854 cims.nyu.edu/~oneil oneil@cims.nyu.edu 212-998-3125
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Course website	https://oneilm.gitlab.io/rnla2022

Prerequisites
Graduate-level proficiency in multivariate calculus, probability theory, linear algebra. Programming experience recommended, but not required.

Description
Numerical linear algebra has been the bedrock of computational science for the past several decades. The ability to solve linear systems, find eigenvalues/eigenvectors, and compute matrix factorizations has allowed many problems in the physical sciences to be solved quite accurately (even nonlinear ones, as often the nonlinear term can be iterated on in order to solve the overall system). However, as the dimension of problems grows, standard dense numerical linear algebra algorithms will fail to keep up due to their inherent computational complexity, often quadratic or cubic in the dimension of the matrix. Recently, methods of randomization have been introduced which can partially overcome this computational complexity growth, oftentimes even avoiding a significant loss in precision. This course will serve as an overview of this class of randomized techniques, how they work in practice, the underlying analysis of accuracy guarantees, and modern applications of such techniques to problems in math, physics, chemistry, and statistics.

Materials
There is no textbook for the course. Course materials will consist of a selection of journal articles and lecture notes. A solid foundation in linear algebra (and numerical linear algebra) is recommended, and we suggest the following sources with regard to those topics:

- L. N. Trefethen and D. Bau, *Numerical Linear Algebra*, SIAM, 1997
- P. Lax, *Linear Algebra and its Applications*, Wiley, 2007

Assignments and grading

There will be a final presentation in the course, consisting of a project or a detailed presentation of a relevant journal paper. The final grade will be determined based on this presentation and class participation.

Weekly schedule

1. Numerical linear algebra refresher, computational complexity, efficient algorithms
2. Randomized projections for low-rank approximation
3. Error estimation and adaptivity
4. Randomized sampling schemes
5. Fast algorithms
6. Applications Part 1
7. Refresher: Iterative linear algebra
8. Iterative sketching
9. Sparsification: Inversion
10. Sparsification: Eigenproblems
11. Variational Monte Carlo
12. Applications Part 2
13. Optional topics
14. Optional topics
15. Presentations

Academic Integrity

This course will adhere to the academic integrity statement issued by the NYU Graduate School of Arts and Sciences, available at:
https://gsas.nyu.edu/content/nyu-as/gsas/about-gsas/policies-and-procedures/gsas-statement-on-academic-integrity.html